Strong coupling constant from bottomonium fine structure.

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Abstract

From a fit to the experimental data on the $b\bar{b}$ fine structure, the two-loop strong coupling constant is extracted. For the $1P$ state the fitted value is $\alpha_s(\mu_1) = 0.33 \pm 0.01\,(\mathrm{exp}) \pm 0.02\,(\mathrm{th})$ at the scale $\mu_1 = 1.8 \pm 0.1$ GeV, which corresponds to the QCD constant $\Lambda^{(4)}\,(2-\mathrm{loop}) = 338 \pm 30$ MeV $(n_f = 4)$ and $\alpha_s(M_Z) = 0.119 \pm 0.002$. For the $2P$ state the value $\alpha_s(\mu_2) = 0.40 \pm 0.02\,(\mathrm{exp}) \pm 0.02\,(\mathrm{th})$ at the scale $\mu_2 = 1.02 \pm 0.02$ GeV is extracted, which is essentially larger than in the previous analysis of refs. [4, 5], but about 30% smaller than the value given by the standard perturbation theory. This value $\alpha_s(1.0) \approx 0.40$ can be obtained in the framework of the background perturbation theory thus demonstrating the freezing of $\alpha_s(\mu)$. The relativistic corrections to α_s are found to be about 15%.

PACS 11.10.Ij,11.15Bt,12.38.Lg

1 Introduction

The bottomonium spectrum is one of the richest among all known mesons and its levels were measured with high precision [1]. These data about $b\bar{b}$ states have been intensively studied in different theoretical approaches, in particular, to determine the QCD strong coupling constant $\alpha_s(\mu)$ at different energy scales μ from the level differences [2-10]. At present, however, there is no clear picture which are the exact values of $\alpha_s(\mu)$ for the $b\bar{b}$ levels and how they are changing from the ground state to the excited ones. There are several reasons for that.

First of all, there is no experimental information on the $\eta_b(nS)$ masses and therefore $\alpha_s(\mu)$ cannot be directly determined from the $b\bar{b}$ hyperfine splittings in S-wave states.

Second, to describe the fine structure splittings in the P-wave states, different energy scales μ were used in different theoretical analyses [4-7]. In ref. [4] $\alpha_s(\mu) = 0.33$ ($\mu = 3.25$ GeV) was taken for all $b\bar{b}$ S- and P-wave states, while in [5] μ was chosen to be equal to the b quark mass, $\mu = m$ with either $m = 4.6$ GeV or $m = 5.2$ GeV. The fitted values of $\alpha_s(\mu)$ were found to be $\alpha_s(m) = 0.22 \div 0.27$ [5] and for the $2P$ state $\alpha_s(\mu)$ appeared to be smaller than for the $1P$ state.

An important step to clarify this problem was taken in [6, 7] where the low-lying bottomonium states, $1S$, $2S$, and $1P$ were investigated. It was observed there that the scale μ is a decreasing function of the principal quantum number n,$\mu = 2(na)^{-1}$ where a is a Coulomb type radius. Therefore, μ is found to be equal for the $2S$ and $1P$ states and the values $\mu = 1.7GeV$, $\alpha_s(1.7) = 0.29$, were determined from the fine structure splittings of $\chi_b(1P)$. Also, $\alpha_s(\mu)$ is larger for excited states with a larger radius of the system, thus indicating that for a bound state the characteristic scale μ is determined by the size, but not by the momentum of the system. One of our main goals here is to check this important statement for the $2P$ state, $\chi_b(2P)$, which cannot be studied in the framework of the approach developed in [6, 7].

In the present study of the $1P$ and $2P$ $b\bar{b}$ states we shall try to answer the following questions:

What are the values of $\alpha(\mu)$ for the $2P$ and the $1P$ states?

Do the extracted values of $\alpha_s(\mu)$ correspond to the existing experimental data on $\alpha_s(M_Z)$ and $\Lambda^{(n_f)}$?

How does $\alpha_s(\mu)$ depend on the relativistic corrections to the wave functions in bottomonium?

How sensitive are the extracted values of $\alpha_s(\mu)$ to the b quark pole mass and the parameters of the static interaction?

2 Perturbative Radiative Corrections

It is well known that one cannot describe the fine structure splittings in heavy quarkonia without taking into account the second order radiative corrections [4-7, 10]. In coordinate space, perturbative static and spin-dependent potentials in the $\overline{MS}$ renormalization scheme were obtained in [2, 3]. From the potentials given there one can immediately find the matrix elements of the spin-orbit and the tensor potentials: $a = \langle V_{LS}(r)\rangle$, $c = \langle V_T(r)\rangle$. Below we give their expressions for a number of flavours $n_f = 4$, valid for the $b\bar{b}$ system:

$$a_{\rm P} = a_{\rm P}^{(1)} + a_{\rm P}^{(2)}, \tag{1}$$

$$a_{\rm P}^{(1)} = \frac{2\alpha_s(\mu)}{m^2}\langle r^{-3}\rangle, \quad a_{\rm P}^{(2)} = \frac{2\alpha_s^2(\mu)}{\pi m^2}\left\{\langle r^{-3}\rangle\left(\frac{25}{6}\ln\left(\frac{\mu}{m}\right) + A\right) + \frac{13}{6}\langle r^{-3}\ln mr\rangle\right\} \tag{2}$$

and for the perturbative part of the the tensor splitting $c_{\rm P}$,

$$c_{\rm P} = c_{\rm P}^{(1)} + c_{\rm P}^{(2)}, \tag{3}$$

$$c_{\rm P}^{(1)} = \frac{4}{3}\,\frac{\alpha_s(\mu)}{m^2}\langle r^{-3}\rangle, \quad c_{\rm P}^{(2)} = \frac{4}{3}\,\frac{\alpha_s^2(\mu)}{\pi m^2}\left\{\langle r^{-3}\rangle\left(\frac{25}{6}\ln\frac{\mu}{m} + B\right) + \frac{7}{6}\langle r^{-3}\ln mr\rangle\right\}. \tag{4}$$

Here the constant $A = \frac{13}{6}\gamma_E + \frac{7}{36} = 1.44508$ and $B = \frac{7}{6}\gamma_E + \frac{33}{12} = 3.42342$.

For our analysis it is convenient to introduce a linear combination of the matrix elements a and c as was done in ref. [10]: $\eta = \frac{3}{2}c - a$. Its perturbative part $\eta_{\rm P}$ is

$$\eta_{\rm P} = \frac{3}{2}c_{\rm P} - a_{\rm P} = \frac{3}{2}c_{\rm P}^{(2)} - a_{\rm P}^{(2)} = \frac{2\alpha_s^2(\mu)}{\pi m}f_4. \tag{5}$$

The factor f_4 in Eq. (5) can be found from Eqs. (2) and (4),

$$f_4(nP) = \frac{1}{m}\left[1.97834\,\langle r^{-3}\rangle_{nP} - \langle r^{-3}\ln mr\rangle_{nP}\right]. \tag{6}$$

For the fine structure analysis it turns out to be very important that the combination of matrix elements f_4 does not depend on the energy scale μ. Later, it will be also shown that f_4 has the largest relativistic correction (about 35%) compared to other matrix elements and depends weakly on the parameters of the static interaction and on the mass of the b-quark.

3 Nonperturbative Contributions

Besides the perturbative terms Eqs. (2, 4) the tensor and spin-orbit splittings have in general nonperturbative contributions: $a = a_{\rm P} + a_{\rm NP}$ and $c = c_{\rm P} + c_{\rm NP}$.

The nonperturbative part of the spin-orbit parameter $a_{\rm NP}$ is determined by the chosen confining potential. Here we shall take a linear potential σr at all distances. The linear behaviour of the confining potential at large distances is well established phenomenologically due to the existence of the meson Regge trajectories and was also deduced from the minimal area law for Wilson loops in QCD (see the review [11] and the references therein). Recently, the linear behaviour of the confining potential was found at small distances due to the interferences of perturbative and nonperturbative (NP) effects [12] and to the saturation property of the QCD strong coupling constant in vacuum fields [13-15]. For the linear potential σr the nonperturbative interaction is given by the Thomas potential for which

$$a_{\rm NP} = -\frac{\sigma}{2m^2}\langle r^{-1}\rangle \tag{7}$$

The nonperturbative contribution to the tensor splitting can be found from the vacuum field correlator $D_1(x)$ [10, 16] which was measured in lattice QCD [17] and was found to be of exponential form. Then, as was shown in [10, 18, 19],

$$c_{\rm NP} = \frac{D_1(0)}{3m^2 T_g}\langle r^2 K_0(r/T_g)\rangle \equiv \frac{D_1(0)}{3m^2} J\ (T_g), \quad J(T_g) \equiv \frac{1}{T_g}\langle r^2 K_0(r/T_g)\rangle, \tag{8}$$

where T_g is the vacuum correlation length. Lattice QCD without dynamical fermions give $T_g \approx 0.2$ fm and $T_g \approx 0.3$ fm in the presence of dynamical fermions with four flavours[17].

In refs. [17] the correlator $D_1(0)$ in Eq. (8) was shown to be small: lattice calculations in quenched SU(3) theory give $D_1(0)/D(0) \approx \frac{1}{3}$ and in full QCD with four staggered fermions $D_1(0)/D(0) \approx 0.1$, where $D(0)$ is another vacuum field correlator which mostly determines the confining potential. These two correlators at the point $x = 0$ can be expressed through the vacuum gluonic condensate G_2 (here the vacuum correlators are normalized as in [11, 16]):

$$D(0) + D_1(0) = \frac{\pi^2}{18}\, G_2. \tag{9}$$

Therefore, the estimate for $D_1(0)/D(0)$ is $0.1 \div 0.3$ and from the relation (9) one obtains

$$\frac{\pi^2}{180} G_2 \lesssim D_1(0) \lesssim \frac{\pi^2}{72}\, G_2. \tag{10}$$

Our calculations give the following typical values for the matrix elements $J(T_g)$: for the $1P$ state $J(T_g) \approx 0.17$ GeV^{-1} and for the $2P$ state $J(T_g) \approx 0.20$ GeV^{-1}, if the b quark mass $m \approx 4.8$ GeV and $T_g \approx 0.2 \div 0.3$ fm is taken. Then, if the value of the gluonic condensate $G_2 = 0.05 \pm 0.02$ GeV^4 [7] is used, one finds the estimate in quenched QCD

$$c_{\rm NP} \lesssim 0.03 \pm 0.01 \text{ MeV}. \tag{11}$$

In full QCD an even smaller value is found. This value of $c_{\rm NP}$ is much less than both $|a_{\rm NP}|$, Eq. (7), and the experimental errors. Therefore it can be neglected in the tensor splitting c and also in $\eta_{\rm NP} = \frac{3}{2}c_{\rm NP} - a_{\rm NP}$, i.e.we take here $\eta_{\rm NP} = a_{\rm NP}$.

4 Fitting Conditions

To fit the experimental data

$$\begin{aligned} a_{\rm exp}(1P) &= 14.23 \pm 0.53 \text{ MeV},\ a_{\rm exp}(2P) = 9.39 \pm 0.18 \text{ MeV}, \\ c_{\rm exp}(1P) &= 11.92 \pm 0.25 \text{ MeV},\ c_{\rm exp}(2P) = 9.14 \pm 0.25 \text{ MeV}, \end{aligned} \tag{12}$$

the following conditions have to be satisfied:

$$\begin{aligned} a_{\rm tot}(nP) &= a_{\rm P}^{(1)} + a_{\rm P}^{(2)} - \frac{\sigma}{2m^2}\langle r^{-1}\rangle = a_{\rm exp}(nP) \\ c_{\rm tot}(nP) &= c_{\rm P}^{(1)} + c_{\rm P}^{(2)} = c_{\rm exp}(nP). \end{aligned} \tag{13}$$

As seen from Eqs. (2) and (4), the l.h.s. of these expressions strongly depend on the normalization scale μ, but the combination η does not. The fitting condition for η is

$$\eta(nP) = \frac{2\alpha_s^2(\mu)}{\pi m} f_4 + \frac{\sigma}{2m^2}\langle r^{-1}\rangle = \eta_{\rm exp}, \tag{14}$$

where the experimental values for $\eta_{\rm exp}^{(nP)}$ are

$$\eta_{\rm exp}(1P) = 3.65 \pm 0.9 \text{ MeV}, \quad \eta_{\rm exp}(2P) = 4.32 \pm 0.4 \text{ MeV}. \tag{15}$$

The condition (14) does not depend explicitly on μ and can be rewritten as

$$\frac{2\alpha_s^2(\mu)}{\pi m} f_4(nP) = \eta_{\rm exp} - \frac{\sigma}{2m^2}\langle r^{-1}\rangle \equiv \Delta(nP) \tag{16}$$

hence the strong coupling constant can be expressed as

$$\alpha_s(\mu) = \sqrt{\frac{\pi m \Delta}{2 f_4}}. \tag{17}$$

For a chosen interaction and quark mass m, $\Delta(nP)$ and $f_4(nP)$ are known numbers and one can immediately determine $\alpha_s(\mu)$.

The extraction of $\alpha_s(\mu)$ from the condition (17), in general, extremely simplifies the fit and also puts strong restrictions on the possible choice of the normalization scale μ. Just this condition was exploited in [10] to determine $\alpha_s(\mu)$ for the $1P$ state in charmonium. In charmonium $\eta_{exp} \approx 24$ MeV and the typical value of $\Delta \approx 7 \div 8$ MeV is not small, so the uncertainty in the extracted value of $\alpha_s(\mu)$ is about 10%.

In bottomonium the typical values of $|a_{\rm NP}|$ are found to be smaller: $|a_{\rm NP}(1P)| = 2.6 \pm 0.2$ MeV (see Table 5) and $|a_{\rm NP}(2P)| = 1.95 \pm 0.10$ MeV (see Table 4). As a result, the numerical values of $\Delta(nP)$ to be substituted in Eq. (17) are small:

$$\Delta(1P) = 1.05 \pm 0.9(\text{exp}) \pm 0.15(\text{th}) \text{ MeV}, \quad \Delta(2P) = 2.4 \pm 0.4(\text{exp}) \pm 0.10(\text{th}) \text{ MeV}. \tag{18}$$

The theoretical uncertainties in this equation are caused by the uncertainty of the value of $a_{\rm NP}$ in the Thomas interaction. Still, for the $2P$ state the total error in $\Delta(2P)$ is not large, about 20%, and therefore $\alpha_s(\mu)$, proportional to $\sqrt{\Delta}$, can be determined from the condition (17) with an accuracy of about 10%. Our calculations show also that the matrix element f_4 in Eq. (17) is practically constant and therefore the theoretical error in Eq. (18) coming from f_4 is small.

For the $1P$ state the experimental error in $\eta_{\rm exp}$, Eq. (15), as well as in Δ, Eq. (18), is large: it comes mostly from the experimental uncertainty in the $\chi_{b_0}(1P)$ mass. Therefore $\Delta(1P)$ can vary in a wide range: $0 \leq \Delta \leq 2.0$ MeV and the relation (17) cannot give an accurate value for $\alpha_s(\mu)$. Instead, for the $1P$ state one needs to use the conditions (13) which are μ-dependent and less restrictive.

5 Dependence on Scale

The second-order perturbative corrections to the spin-orbit and tensor splittings, which are not small, explicitly depend on the scale μ. In Eqs. (2, 4) $\ln(\mu/m)$ enters with the large coefficient 25/6 and therefore the choice $\mu = m$ (causing this logarithm to vanish) can give rise to inconsistent results. Just this choice was taken in [5] where two b-quark masses, $m = 4.6$ GeV and $m = 5.2$ GeV, were analysed. We shall discuss here some results of ref. [5].

From the fit in [5] it was obtained that the value $\tilde{\alpha_s}(m)$ extracted from the tensor and the spin-orbit splittings are slightly different and for the $2P$ state this difference is increasing. (Here $\tilde{\alpha}(\mu)$ or $\tilde{\alpha}(\tilde{\mu})$ denotes the fitted (extracted) value of the strong coupling constant.)

Also, for the $2P$ state $\tilde{\alpha}(5.2) = 0.26 \pm 0.01$ is a bit larger than $\tilde{\alpha}_s(4.6) = 0.25 \pm 0.01$ for the smaller b-quark mass, in contradiction with the standard behaviour of the running coupling constant $\alpha_s(q^2)$.

The extracted value, $\tilde{\alpha}_s(m) \approx 0.25 \div 0.27$, turned out to be about 20% larger than the values $\alpha_s(4.6)$ and $\alpha_s(5.2)$ calculated with the conventional value of $\Lambda^{(4)}$ Eq. (19): $\alpha_s(4.6) = 0.22 \pm 0.01$, $\alpha_s(5.2) = 0.21 \pm 0.01$.

In the calculations that follow, it will be easy to compare our results with those from ref. [5] because in both cases the same perturbative interaction and linear potential σr were used. However, the calculations of ref. [5] were done in the nonrelativistic case (for fixed $\sigma = 0.2$ GeV^2 and two b-quark masses). Here both relativistic and nonrelativistic cases will be considered and σ, m, and α_{eff} of the Coulomb potential will be varied in a wide range. From our analysis it will be clear that the inconsistencies in the $\tilde{\alpha}_s(\mu)$ behaviour mentioned above, are related to the a priori choice $\mu = m$ made in [5].

At this point it is worthwhile to note that at present the QCD constant $\Lambda^{(n_f)}$ is well known for $n_f = 5$, because $\alpha_s(M_z) = 0.119 \pm 0.002$ is established from different experiments: $\Lambda^{(5)}(2 - \text{loop}) = 237^{+26}_{-24}$ MeV and $\Lambda^{(5)}(3 - \text{loop}) = 219^{+25}_{-23}$ MeV are given in [1]. Then from the matching of $\alpha_s(\mu)$ at the scale $\mu = \bar{m}_b$ ($\bar{m}_b$ is the running mass in the $\overline{MS}$ scheme) and taking $\bar{m}_b = 4.3 \pm 0.2$ GeV [1] one can find $\Lambda^{(4)}(3 - \text{loop}) = 296^{+31}_{-29}$ MeV or in the two-loop approximation $\Lambda^{(4)}$ is

$$\Lambda^{(4)}(2 - \text{loop}) = 338^{+33}_{-31} \text{ MeV}. \tag{19}$$

It is of interest to compare $\alpha_s(\mu)$ for $\Lambda^{(4)}$ given by Eq. (19) with the fitted values $\tilde{\alpha}_s(\tilde{\mu})$ used in different theoretical analyses: $a_s(3.25) = 0.251 \pm 0.009$ whereas in [4] the fitted value $\tilde{\alpha}_s(3.25) = 0.33$; $\alpha_s(4.60) = 0.221 \pm 0.007$ while in [5] $\tilde{\alpha}_s(4.6) \approx 0.27$. In both fits the extracted values appeared to be about 20% larger.

This 20% difference implies either very large values of $\Lambda^{(4)}$ or an essentially smaller scale of μ. For example, $\alpha_s(\mu_0) = 0.33$ with the conventional $\Lambda^{(4)}$, Eq. (19), corresponds to $\mu_0 = 1.80 \pm^{0.18}_{0.16}$ GeV instead of $\tilde{\mu} = 3.25$ GeV in [4] and this μ_0 would be in good agreement with the one cited in [6, 7] and with our result (see Section 9).

In our present analysis when different sets of parameters are taken, we shall impose two additional restrictions:

1) For the given P-state the extracted value of $\tilde{\alpha}_s(\mu)$ must be the same for the tensor and the spin-orbit splittings, because both interactions have the same r^{-3} behaviour and they also have the same characteristic size (momentum).

2) Only those sets of parameters for which the fitted two-loop value of $\tilde{\alpha}(\mu)$ corresponds to the conventional value of $\Lambda^{(4)}$ in two-loop approximation, Eq. (19), are deemed appropriate.

6 Static Potential

In heavy $Q\bar{Q}$ systems the spin-dependent interaction contains the factor m^{-2} and therefore it is small and can be considered as a perturbation. For the unperturbed Hamiltonian we considered two cases,relativistic and nonrelativistic,

$$H_0^R = 2\,\sqrt{\vec{p}^2 + m^2} + V_{\rm st}(r) \tag{20}$$

or

$$H_0^{NR} = \frac{\vec{p}^2}{m} + V_{\rm st}(r). \tag{21}$$

Here a static potential, $V_{\rm st}(r) = V_{\rm st}^P(r) + V_{\rm st}^{NP}(r)$, needs some remarks. The perturbative static potential is now known in two-loop approximation [20], but for our discussion it is enough to take it in one-loop approximation from [3]:

$$V_{\rm st}^P = -\frac{4}{3}\frac{\alpha_V(r)}{r} \tag{22}$$

Here the vector coupling constant $\alpha_V(r)$ is expressed through $\alpha_s(\mu)$ in the $\overline{MS}$ scheme in the following way [3]: $(\alpha_s(\mu) << 1)$

$$\begin{aligned}\alpha_V(r) &= \alpha_s(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(a_1 + \frac{\beta_0}{2}\,(\ln(\mu r) + \gamma_E)\right)\right] \\ &= \frac{\alpha_s(\mu)}{1 - \frac{\alpha_s(\mu)}{\pi}\left(a_1 + \frac{\beta_0}{2}(\ln(\mu r) + \gamma_E)\right)} \to \frac{4\pi}{\beta_0 \ln((\Lambda_R r)^{-2})}.\end{aligned} \tag{23}$$

In Eq. (23) we have used $\alpha_s(\mu) = 4\pi/[\beta_0 \ln(\mu^2/\Lambda^2_{\overline{MS}})]$, and the conventional QCD constant in coordinate space: $\Lambda_R = \Lambda_{\overline{MS}} \exp(\gamma_E + a)$ where $a = 2a_1/\beta_0$. We see that the dependence on μ disappears. The constants are: $\beta_0 = 11 - 2n_f/3$, so for $n_f = 4$, $\beta_0 = 25/3$; $a_1 = 31/12 - 5n_f/18$, so for $n_f = 4$, $a_1 = 53/36$.

This expression is valid only for small radiative corrections or small distances: $re^{\gamma_E}\tilde{\Lambda}^{(4)} \ll 1$ or $r \ll 2$ GeV^{-1} = 0.4 fm ($\tilde{\Lambda}^{(4)} \approx 0.3$ GeV). However, in bottomonium the sizes of the different states are varying in a wide range, e.g., typical values of the root-mean-square radius, $R(nL) = \sqrt{\langle r^2\rangle_{nL}}$, are:

$$\begin{aligned}R(1S) &= 0.2 \text{ fm},\ R(1P) = 0.4 \text{ fm},\ R(2S) = 0.5 \text{ fm}, R(2P) = 0.65 \text{ fm}, \\ R(3S) &= 0.7 \text{ fm},\ R(3P) = 0.85 \text{ fm},\ R(4S) = 0.9 \text{ fm}.\end{aligned} \tag{24}$$

These numbers are practically independent of the choice of the static potential parameters and the confining potential provided the chosen potential reproduces the bottomonium spectrum with good accuracy.

From Eq. (24) one can see that the sizes of the nL states run from 0.2 fm to 0.9 fm. Therefore the perturbative potential, Eq. (23), valid for $r \ll 0.4$ fm, can be used only for low-lying states. For the $1S$, $2S$, and $1P$ states this perturbative interaction (also with two-loop corrections) was analyzed in detail in refs. [6, 7] and there it was found that (i) for the $1S$ and $2S$ states the values of μ are different and (ii) μ is smaller in the $2S$ state.

Therefore, one can expect that for every level a specific consideration is needed to determine μ or $\alpha_s(\mu)$.

To describe the $2P$ state, the size of which is about 0.65 fm, or the $b\bar{b}$ spectrum as a whole, a different approach is needed. Here we suggest instead of the perturbative potential Eq. (22) to use the perturbative potential in backgroundd vacuum fields, $V_B(r)$:

$$V_B(r) = -\frac{4}{3}\,\frac{\alpha_B(r)}{r}, \tag{25}$$

in momentum space

$$V_B(q^2) = -\frac{4}{3}\,\frac{4\pi}{\vec{q}^2}\,\tilde{\alpha}_B(q^2), \quad q^2 \equiv \vec{q}^2. \tag{26}$$

In this potential $\tilde{\alpha}_B(q^2)$ is a vector coupling constant in vacuum background fields which was introduced in [14] and applied to $e^+e^- \to$ hadrons processes in [21]:

$$\tilde{\alpha}_B(q^2) = \frac{4\pi}{\beta_0 t_B}\left[1 - \frac{\beta_1 \ln t_B(q)}{\beta_0^2 t_B(q)}\right], \quad t_B(q) = \ln\frac{q^2+m_B^2}{\tilde{\Lambda}^2}, \tag{27}$$

with $\beta_0 = 25/3$. For the vector coupling constant, $\alpha_V(q^2)$, $\tilde{\Lambda}$ differs from Λ in the $\overline{MS}$ scheme: $\tilde{\Lambda} = \Lambda^{(4)}_{\overline{MS}}\, e^a = 481^{+47}_{-41}$ MeV, $a = \frac{5}{6} - \frac{4}{\beta_0} = 0.35333$, and $\Lambda^{(4)}_{\overline{MS}}$ was taken from Eq. (19). (In the $\overline{MS}$ scheme Λ_B and $\Lambda_{\overline{MS}}$ coincide for $n_f = 4, 5$ because of their identical behaviour at large q^2 [10].) The background mass m_B was found from the fit to the charmonium fine structure in [10] where $m_B = 1.1$ GeV was obtained.

In coordinate space $\alpha_B(r)$ can be calculated from the Fourier transform of the potential Eq. (26) with $\alpha_B(q^2)$ given by Eq. (27). Then

$$\alpha_B(r) = \frac{8}{\beta_0}\int_0^\infty dq\,\frac{\sin qr}{q t_B(q)}\left[1 - \frac{\beta_1}{\beta_0^2}\,\frac{\ln t_B(q)}{t_B(q)}\right]. \tag{28}$$

The strong coupling constant in vacuum background fields maintains the property of asymptotic freedom at small r, $r \ll \tilde{\Lambda}^{-1}$ and $r \ll m_B^{-1}$,

$$\alpha_B(r \to 0) = -\frac{2\pi}{\beta_0 \ln(\tilde{\Lambda} e^{\gamma} r)} \tag{29}$$

Here the function $\gamma = \gamma(r)$ is

$$\gamma = \gamma(r) = \gamma_E + \Sigma, \quad \Sigma = \sum_{k=1}^{\infty}\frac{(-m_B r)^k}{k!k}, \tag{30}$$

or at small r

$$\gamma = \gamma_E - m_B r, \tag{31}$$

whereas in standard perturbative theory $\gamma^P = \gamma_E = 0.5772$. Due to the dependence on the distance r in Eq. (31) the expression Eq. (29) is always bounded.

For large r^2, $r^2 \gg m_B^{-2}$, the limit of $\alpha_B(r)$ in Eq. (28) tends to a constant, denoted as $\alpha_B(\infty)$ and called the freezing value:

$$\alpha_B(\infty) = \frac{4\pi}{\beta_0 t_0}\left[1 - frac\beta_1 \ln t_0 \beta_0^2 t_0\right], \qquad t_0 = \ln\frac{m_B^2}{\tilde{\Lambda}^2}. \tag{32}$$

From the integral Eq. (28) it can be shown that the freezing value is the same in coordinate and in momentum space, $\alpha_B(r \to \infty) = \tilde{\alpha}_B(q^2 = 0)$. The properties of $\alpha_B(r)$ were discussed in [10, 13, 14] and a detailed analysis of $\alpha_B(r)$ will be published elsewhere.

In the intermediate region, 0.2 fm $\leq r \leq$ 0.9 fm, $\alpha_B(r)$ approaches rapidly the value $\alpha_B(\infty)$.

Therefore, to study the bottomonium spectrum as a whole it is convenient to introduce an effective constant α_{eff}:

$$\alpha_B(r) = \alpha_{\text{eff}} + \delta\alpha_B(r), \quad \alpha_{\text{eff}} = \text{constant}, \qquad |\delta\alpha_B(r)| \ll \alpha_{\text{eff}}, \tag{33}$$

and to consider the contribution from the term $\delta V_B(r)$,

$$\delta V_B(r) = -\frac{4}{3}\,\frac{\delta\alpha_B(r)}{r}, \tag{34}$$

as a perturbation. Then in the Hamiltonian (22) the static interaction

$$V_0(r) = -\frac{4}{3}\,\frac{\alpha_{\text{eff}}}{r} \tag{35}$$

will be taken into account as an unperturbed interaction.

For the nonperturbative interaction a linear form σr will be taken and therefore the static potential in the unperturbed Hamiltonian $V_0(r)$,

$$V_0(r) = -\frac{4}{3}\,\frac{\alpha_{\text{eff}}}{r} + \sigma r + C_0 \tag{36}$$

coincides with the well known Cornell potential. Later, the values of the string tension σ will be varied in the range $0.17 \div 0.20$ GeV^2.

We shall present a detailed analysis of the $b\bar{b}$ spectrum in a separate paper.

7 Relativistic Corrections

There exists the point of view that in bottomonium the relativistic corrections are small because of the heavy b quark mass. Indeed, the comparison of levels and mass differences for the Schrödinger equation and the Salpeter equation, Eqs. (20,21), in general, confirms this statement (here the static potential is supposed to be the same in both cases). In Table 1 the $b\bar{b}$ mass differences are given for two typical sets of parameters.

From Table 1 one can see that

(i) Relativistic corrections are small for large mass differences like $M(n, L) - M(n-1, L)$ or $M(nL) - M(n, L-1)$;

(ii) For close lying levels, like $\Delta_1 = M(2S) - M(1P)$ and $\Delta_2 = M(3S) - M(2P)$, the corrections are essential, about 15%, and to get Δ_1 and Δ_2 close to the experimental data it is necessary to take into account the contribution from the perturbation $\delta V_B(r)$ Eq. (34). In the relativistic case the influence of the phenomenon of asymptotic freedom appears to be more essental than in the nonrelativistic (NR) case.

Table 1: Bottomonium level differences (MeV) for the Schrödinger and the Salpeter equations.

Mass differences	Set I, $\alpha_{\text{eff}} = 0.3545$ $m = 4.737$ GeV $\sigma = 0.20$ GeV^2 (^{a})		Set II, $\alpha_{\text{eff}} = 0.36$ $m = 4.81$ GeV $\sigma = 0.18$ GeV^2		Exp. val. (MeV)
	Rel.	Nonrel.	Rel.	Nonrel.	
$M(2S) - M(1S)$	554.34	551.97	556.55	550.03	562.9 ± 0.5
$M(3S) - M(2S)$	350.43	354.78	335.62	338.49	332.0 ± 0.8
$M(4S) - M(3S)$ (^{b})	285.93	291.83	270.63	275.30	224.7 ± 4.0
$M(1P) - M(1S)$	458.04	439.66	473.49	450.15	439.8 ± 0.9
$M(2P) - M(1P)$	359.67	366.75	342.55	348.70	359.8 ± 1.2
$M(2S) - M(1P)$	96.31	112.31	83.07	99.88	123.1 ± 1.0
$M(3S) - M(2P)$	87.06	100.34	72.82	89.67	95.3 ± 1.0

(^{a}) This set was taken from ref. [22]
(^{b}) The $4S$ level lies above the $B\bar{B}$ threshold

Table 2: $1P$-state matrix elements for the Schrödinger and the Salpeter equations.

Matrix element	Set I (^{a})		Set II (^{a})	
	Rel.	Nonrel.	Rel.	Nonrel.
$\sqrt{\langle r^2 \rangle}$ (GeV^{-1})	1.994	2.039	2.008	2.054
$\langle r^{-1} \rangle$ (GeV)	0.633	0.614	0.631	0.612
$\langle r^{-3} \ln mr \rangle$ (GeV^3)	0.675	0.631	0.681	0.636
$\langle r^{-3} \rangle$ (GeV^3)	0.551	0.483	0.556	0.485
$f_4(1P)$ (GeV^2)	0.0876	0.0685	0.0871	0.0673

(^{a}) For the parameters see Tab. 1

Table 3: $2P$-state matrix elements for the Schrödinger and the Salpeter equations.

Matrix element	Set I (^{a})		Set II (^{a})	
	Rel.	Nonrel.	Rel.	Nonrel.
$\sqrt{\langle r^2 \rangle}$ (GeV^{-1})	3.177	3.263	3.235	3.320
$\langle r^{-1} \rangle$ (GeV)	0.477	0.455	0.469	0.448
$\langle r^{-3} \ln mr \rangle$ (GeV^3)	0.495	0.448	0.489	0.443
$\langle r^{-3} \rangle$ (GeV^3)	0.504	0.414	0.496	0.406
$f_4(1P)$ (GeV^2)	0.1060	0.0783	0.1025	0.0748

(^{a}) For the parameters see Tab. 1

The relativistic corrections are becoming essential for some matrix elements, which determine the fine structure splittings (see Table 2. To calculate them in the relativistic case (for the Salpeter equation) the expansion of the wave function in a series over Coulomb-type functions was used as it was suggested in [22]. The numbers obtained have a computational error $\lesssim 10^{-4}$ (the dimension of the matrices D was varied from D=20 to D=40).

From the numbers given in Table 2) one can conclude that

- for $1P$ and $2P$ states the root-mean-square radii practically coincide in the relativistic and the NR cases;
- for the matrix element $\langle r^{-1}\rangle$ the difference between both cases is small, about 3% for the $1P$ state and about 5% for the $2P$ state; in the relativistic case $\langle r^{-1}\rangle$ and therefore $|a_{\rm NP}(nP)|$ is slightly larger.
- in the relativistic case the values of $\langle r^{-3}\ln\ mr\rangle$ are about 7% (10%) larger for the $1P(2P)$ state for given set of chosen parameters;
- for the Salpeter equation the matrix element $\langle r^{-3}\rangle$ is larger by about 14% (22%) for the the $1P(2P)$ state;
- the largest relativistic correction was found for the factor f_4 given in Eq. (6). This difference is about 30% for the $1P$ state and 36% for the $2P$ state. The numbers given do practically not change for different sets of parameters. So our averaged value of $f_4(nP)$ ($\alpha_{\rm eff} \geq 0.35$) are:

$$f_4(1P) = 0.085 \pm 0.010 \ \ {\rm GeV}^2, \quad f_4(2P) = 0.106 \pm 0.008 \ \ {\rm GeV}^2. \tag{37}$$

The theoretical error in Eq. (37) ($\approx 10\%$) mostly comes from the variation of the b quark mass (in the range $4.6 \div 5.0$ GeV).

The increasing of $f_4(nP)$ in the relativistic case directly affects the values of $\alpha_s(\mu)$ extracted from the fine structure data because according to Eq. (17)

$$\alpha_s(\mu) = \sqrt{\frac{\pi m \Delta(nP)}{2 f_4(nP)}}, \qquad \Delta(nP) = \eta_{\rm exp}(nP) - |a_{\rm NP}(nP)|, \tag{38}$$

is proportional to $f_4^{-1/2}$ and $\alpha_s(\mu)$ is about 15% *smaller* in the relativistic case. This result obtains both for $1P$ and $2P$ states.

Therefore, below we shall use only matrix elements calculated for the Salpeter equation, in this way taking into account the relativistic corrections. A last remark concerns the choice of the quark pole mass, $m_{\rm pole} = m$ which enters the Salpeter equation [6]. Here we study the spin structure of the χ_b mesons determined by the spin-dependent potentials now known only in one-loop approximation. Therefore the pole mass of the b quark will be taken also in one-loop approximation [23]:

$$m = m_{\rm pole} = \bar{m}(\bar{m}^2)\left\{1 + \frac{4}{3}\frac{\alpha_s(m_{\rm pole})}{\pi}\right\} \tag{39}$$

In Eq. (39) $\bar{m}(\bar{m}^2)$ is a running quark mass in the $\overline{MS}$ renormalization scheme, its value from [1] is $\bar{m} = 4.3 \pm 0.2$ GeV. Then taking $\Lambda^{(4)}$ from Eq. (19) one finds m in the range

$$4.5 \ \text{GeV} \le m \le 5.0 \ \text{GeV} \tag{40}$$

Only values of the mass in this range will be used later in our calculations.

8 $\alpha_s(\mu)$ for the $2P$ State

For the $2P$ state $\alpha_s(\mu)$ can be immediately found from the relation (17) for the chosen static potential with fixed parameters α_{eff}, σ, and m. At first, we shall give an estimate of $\alpha_s(\mu)$ using the following results:

1. The nonperturbative spin-orbit splitting $|a_{\text{NP}}(2P)|$ depends weakly on the choice of the parameters, provided the $b\bar{b}$ spectrum is described with good accuracy

$$|a_{\text{NP}}(2P)| = 1.95 \pm 0.15 \ \text{MeV}. \tag{41}$$

2. In Eq. (15) the experimental error of η_{exp} $(2P)$ is not large and therefore the difference $\Delta(2P)$ Eq. (17) is known with an accuracy of about 20%:

$$\Delta(2P) = \eta_{\text{exp}}(2P) - |a_{\text{NP}}(2P)| = 2.40 \pm 0.04(\text{exp}) \pm 0.15(\text{th}) \ \text{MeV}. \tag{42}$$

3. In our calculations the matrix element $f_4(2P)$ is changing in the narrow range:

$$f_4(2P) = 0.106 \pm 0.008 \ \text{GeV}^2. \tag{43}$$

   Then, from the fitting condition (17) and the numbers given in Eqs. (41)-(43) the lower and upper bounds of $\tilde{\alpha}_s(\mu)$ can be determined,

$$\sqrt{\frac{m}{m_0}}0.37 \le \tilde{\alpha}_s(\mu) \le \sqrt{\frac{m}{m_0}}0.46 \tag{44}$$

   Here a normalization mass, $m_0 = \frac{1}{2}M(\Upsilon(1S)) = 4.73$ GeV , was introduced for convenience. Here and below all numbers were calculated in relativistic case, i.e. for the Salpeter equation.

From the estimates (44) it is clear that for the $2P$ state $\alpha_s(\mu) \approx 0.40$ turns out to be large for any set of the parameters of the static interaction. It is essentially larger than that obtained in ref. [4] where $\alpha_s(3.25) = 0.33$ and in ref. [5] where $\alpha_s(4.6) = 0.26$. In our calculations large values of $\tilde{\alpha}_s(\mu)$ are extracted irrespectively to the value of the scale μ, which is still not fixed.

However, $\alpha_s(\mu)$ in Eq. (44) is varying in a rather wide range and its value is sensitive to small variations of the factors entering the condition (17). The value of $\alpha_s(\mu)$ is *decreasing* if the constant α_{eff} of the static interaction is growing. In our numerical calculations the values of α_{eff} is supposed to be in the range,

$$0.35 \le \alpha_{\text{eff}} < \alpha_B(q^2 = 0) \approx 0.48 \tag{45}$$

with a b quark mass from the condition (40).

With the restriction (45) the fitted values of $\tilde{\alpha}_s(\mu)$ appeared to lie in the narrower range,

$$\tilde{\alpha}_s(\mu) = 0.40 \pm 0.02(\mathrm{th}) \pm 0.04(\mathrm{exp}) \quad (b\bar{b}). \tag{46}$$

Here the experimental error comes from $\eta_{\rm exp}$, Eq. (18), and the theoretical error is due to the variation of $\alpha_{\rm eff}$, m, and σ.

In the extracted value $\tilde{\alpha}_s(\mu)$, Eq. (46), the scale μ is still not specified. To find μ_2 it is better to use the condition $c(2P) = c_{\rm exp}$,(13), for the tensor splitting, because the theoretical uncertainty connected with the nonperturbative contribution to $c(2P)$ is negligible, $c_{\rm NP} <$ 0.05 Mev. This condition (13) turns out to be satisfied for the scale,

$$\mu = \mu_2 = 1.02 \pm 0.02 \text{ GeV}, \tag{47}$$

which has a small theoretical error, while the extracted value of $\tilde{\alpha}(\mu)$, Eq. (46), was determined with an accuracy of 15%.

It is of interest to compare $\tilde{\alpha}(1.0) \approx 0.40$ with the value found in perturbation theory. The scale $\mu_2 \approx 1.0$ GeV is small, less than the running mass of the c quark, $\bar{m}_c = 1.3 \pm 0.2$ GeV [1], therefore $\alpha_s(1.0)$ should be calculated with $\Lambda = \Lambda^{(3)}$ (2-loop), $n_f = 3$. The value of $\Lambda^{(3)}$ can be found using the matching condition at $\mu = \bar{m}_c$ and the value of $\Lambda^{(4)}$ (2-loop), Eq. (19). Then

$$\Lambda^{(3)}(2-\text{loop}) = 384^{+32}_{-30} \text{ MeV} \tag{48}$$

and correspondingly the two-loop strong coupling constant is

$$\alpha_s(1.0) = 0.53^{+0.6}_{-0.5}, \tag{49}$$

which is 30% larger than our fitted value given by Eq. (46). It was suggested in [10] that this decreasing of $\tilde{\alpha}(\tilde{\mu})$ at the scale $\mu_2 = 1$ GeV can be explained by the behavior of $\alpha_B(\mu)$ Eq. (27) in the vacuum background fields, thus demonstrating the phenomenon of freezing of $\alpha_s(\mu)$. In [10], from a fit to the charmonium fine structure, the background mass m_B in Eq. (27) was found to be (in the $\overline{MS}$ renormalization scheme)

$$m_B = 1.1 \text{ GeV}, \quad \Lambda^{(3)}_B(2-\text{loop}) = 400^{+40}_{-50} \text{ MeV} \quad (c\bar{c}). \tag{50}$$

Our extracted value of $\tilde{\alpha}(1.0)$ in Eq. (46) corresponds to the close value of $\Lambda^{(3)}_B$,

$$\Lambda^{(3)}_B(2-\text{loop}) = 420^{+40}_{-30} \text{ MeV} \quad (b\bar{b}). \tag{51}$$

Note also that for the $1P$ state in charmonium the value

$$\tilde{\alpha}(1.0) = 0.38 \pm 0.03(\mathrm{th}) \pm 0.04(\mathrm{exp}) \tag{52}$$

practically coincides with $\tilde{\alpha}(1.0)$ in bottomonium,

$$\tilde{\alpha}(1.0) = 0.40 \pm 0.02(\mathrm{th}) \pm 0.04(\mathrm{exp}), \quad \mu_2 = 1.02 \pm 0.02 \text{ GeV}. \tag{53}$$

This coincidence is not, in our opinion, accidental: both states, the $c\bar{c}$ $1P$ state and the $b\bar{b}$ $2P$ state, have the same size: $R = \sqrt{< r^2 >_{nP}} = 0.62 \div 0.65$ fm. This coincidence of the

Table 4: Fine-structure parameters for the $2P$ $b\bar{b}$ state.

	Set I (^{a})	Set II (^{a})	Set III (^{b})	Exp. val.
$\tilde{\alpha}(\mu_2)$	0.392	0.429	0.386	
μ_2 (GeV)	1.03	1.02	1.03	
$c_{\rm P}^{(2)}$ (MeV)	-2.62	-3.14	-3.35	
$c_{\rm tot}$ (MeV)	9.12	9.11	9.17	9.1 ± 0.2
$a_{\rm NP}$ (MeV)	-2.12	-1.83	-1.80	
$a_{\rm P}^{(1)}$ (MeV)	17.61	18.33	18.77	
$a_{\rm P}^{(2)}$ (MeV)	-6.12	-7.19	-7.52	
$a_{\rm tot}$ (MeV)	9.37	9.32	9.45	9.4 ± 0.2

(^{a}) For the parameters see Tab. 1
(^{b}) $\alpha_{\rm eff} = 0.386$, $\sigma = 0.185$ GeV^2, $m = 5.0$ GeV.

values of $\alpha_s(\mu)$ and of the sizes indicates that for the bound states the scale μ is characterized by the size, but not the momentum, of the system. This result is in agreement with the predictions of refs. [6, 7].

With the use of the fitted values $\tilde{\alpha}_s(\mu_2)$, Eq. (53), the theoretical number obtained for the spin-orbit splitting $a_{\rm tot}$ automatically satisfies the third fitting condition Eq. (13). Calculated numbers of a and c are given in Table 4 for three different sets of parameters. From these numbers one can see that the second order radiative corrections $a_{\rm P}^{(2)}$ and $c_{\rm P}^{(2)}$ are negative and rather large: about 25% for the tensor and 40% for the spin-orbit splittings.

Note that we have met here no difficulty to get a precise description of the tensor and spin-orbit splittings for the $2P$ state, in contrast to the results of ref. [5], where some difficulties have occurred, in our opinion, because of the choice $\tilde{\mu} = m$ (see the discussion in Section 5).

9 $\alpha_s(\mu)$ for the $1P$ State

For the $1P$ state the scale-independent condition (17) cannot be used directly, because the important factor $\Delta(1P)$ in Eq. (17) has a large experimental error. So in this case one needs to use the two μ-dependent conditions Eq. (13) on the splittings a and c.

There exist a lot of variants where these two conditions can be satisfied. However, in many cases the two-loop values $\tilde{\alpha}_s(\mu)$ and μ_1, extracted from those fits, correspond to a very large value of the QCD constant $\Lambda^{(4)}$. Therefore, the additional requirement (21) that $\Lambda^{(4)}$ (2-loop) should have a value in the range 307 MeV $\leq \Lambda(4) \leq$ 371MeV, is necessary. If this restriction is put, then in our calculations the extracted scale μ_1 appears to lie in the narrow range,

$$\mu_1 = 1.80 \pm 0.10 \text{ GeV} \tag{54}$$

and

$$\tilde{\alpha}(\mu_1) = 0.33 \pm 0.01(\text{exp}) \pm 0.02(\text{th}). \tag{55}$$

Table 5: Fine-structure parameters for the $1P$ $b\bar{b}$ state.

	Set I (^{a})	Set II (^{a})	Set III (^{b})	Exp. val.
$\tilde{\alpha}(\mu_2)$	0.335	0.340	0.32	
μ_1 (GeV)	1.80	1.85	1.90	
$c_{\rm P}^{(2)}$ (MeV)	0.96	1.03	0.90	
$c_{\rm tot}$ (MeV)	11.93	11.92	11.91	11.92 ± 0.20
$a_{\rm NP}$ (MeV)	-2.82	-2.45	-2.44	
$a_{\rm P}^{(1)}$ (MeV)	16.46	16.34	16.52	
$a_{\rm P}^{(2)}$ (MeV)	0.12	0.21	0.05	
$a_{\rm tot}$ (MeV)	13.76	14.09	14.12	14.23 ± 0.57

(^{a}) For the parameters see Tab. 1
(^{b}) $\alpha_{\rm eff} = 0.386$, $\sigma = 0.185$ GeV^2, $m = 5.0$ GeV.

Our value for the scale μ_1 turned out to be very close to that determined in refs. [7], but our fitted value of $\alpha_s(\mu_1)$ is about 15% larger than the one found in [7] where $\tilde{\alpha}_s$(3-loop)=0.29 and $\Lambda^{(4)}(3-\text{loop}) = 230$ MeV (or $\Lambda^{(4)}$ (2-loop)= 250^{+90}_{-60} MeV) is smaller than in our fit.

For the $1P$ state it was also observed that if a large value $\sigma = 0.2$ GeV^2 is taken, then it is difficult to reach a consistent description of the tensor and the spin-orbit splittings simultaneously. Therefore here, as well as in the charmonium case [10], the values $\sigma = 0.17 \div 0.185$ GeV^2 are considered as preferable. Also the choice of a relatively large b quark mass,

$$m_b = 4.75 \div 4.9 \text{ GeV}, \tag{56}$$

gives rise to a better fit.

The results of our calculations for the $1P$ state are given in Table 5 from which one can see that the second order corrections $c_{\rm P}^{(2)}$ and $a_{\rm P}^{(2)}$ are relatively small, 8% and 15%, but still remain important for a fit to the experimental data. Also in all good fits the effective Coulumb constant $\alpha_{\rm eff}$ lies between $\tilde{\alpha}(\mu_1)$ and $\tilde{\alpha}_s(\mu_2)$:

$$\tilde{\alpha}(\mu_1) < \alpha_{\rm eff} \leq \tilde{\alpha}(\mu_2). \tag{57}$$

In our analysis $\mu_2(2P)$ is less than $\mu_1(1P)$ and their ratio is almost inversely proportional to the ratio of the radii of these states:

$$\frac{\mu_1(1P)}{\mu_2(2P)} \approx 1.7 \div 1.8; \quad \frac{\sqrt{\langle r^2 \rangle_{2P}}}{\sqrt{\langle r^2 \rangle_{1P}}} = 1.6 \div 1.65. \tag{58}$$

This result is in full agreement with the prediction of refs. [6, 7] about the decrease of the scale with increasing principal quantum number.

10 Conclusion

The precise experimental data on the masses of $\chi_{bJ}(1P)$ and $\chi_{bJ}(2P)$ give a unique opportunity to determine the QCD strong coupling constant at low-energy scales. In our analysis of fine structure splittings we found that:

1. The relativistic corrections which are small for such characteristics as the $b\bar{b}$ levels, radii, and matrix element $\langle r^{-1}\rangle$, are nevertheless essential for the determination of the factor $f_4(nP)$, which is inversely proportional to the extracted value of $\tilde{\alpha}_s^2(\mu)$.

2. From a μ-independent analysis of the $2P$ state, the value $\tilde{\alpha}_s(\mu_2) \approx 0.40$ was extracted. The scale $\mu_2 = 1.0 \pm 0.02$ GeV, determined from the tensor splitting, appeares to be practically unchanged for any chosen set of parameters.

3. The extracted value $\tilde{\alpha}(1.0) \approx 0.40$ is about 30% lower than the one found in perturbation theory if $\Lambda^{(3)} = 384^{+32}_{-30}$ MeV was used. This value agrees with the fitted $\alpha_s(1.0, c\bar{c})$ extracted from the analysis of the charmonium fine structure. This result can be naturally explained in the framework of background perturbation theory.

4. The scale $\mu_1 \approx 1.8$ GeV for the $1P$ $b\bar{b}$ state obtained here agrees with the prediction in [7] but corresponds to the larger value $\Lambda^{(4)}$(2-loop)$=338^{+33}_{-31}$ MeV, which gives rise to $\alpha_s(M_z) = 0.119 \pm 0.002$.

5. The preferred values of the pole mass of the b quark are found to be $m = 4.7 \div 4.9$ GeV but from the fine structure analysis we could not narrow their range.

Our results have confirmed the important observation of Yndurain et al. [6, 7] that the strong coupling constant is increasing for states with a larger size or larger principal quantum number and this fact is essential in many aspects of quarkonium physics.

One of the authors (A.M.B.) acknowledges a partial financial support through the grants RFFI-DFG No.96-02-00088g

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